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Explicit Solution of Difference Equation for the Wavenumber Response of Fluid-Loaded Stiffened Plate

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PREFACE

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A method for solving a difference equation encountered for excitation of a line-driven, fluid-loaded, infinite flat plate with nonperiodic attached rib stiffeners is presented. For the case of excitation by Q complex exponentials, a linear set of Q simultaneous equations must be solved for each wavenumber k. For the most general excitation, a linear integral equation must be solved at each k. A possible shortcut for obtaining solutions at certain shifted values of wavenumber k is pointed out.

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LIST OF SYMBOLS

L	periodic inter rib spacing
Δ	offset of one set of rib stiffeners to another set
Q	number of sets of attached rib stiffeners, (1)
D	rigidity of plate, (1)
$w(x)$	transverse plate displacement, (1)
m	mass per unit area of plate, (1)
ω	applied excitation frequency (radians/second), (1)
$P_e(x)$	external pressure due to line-force, (1)
$P_a(x,0)$	acoustic pressure on upper surface of plate, (1)
$P_q(x)$	total pressure exerted by q-th set of rib stiffeners
k	wavenumber, (2)
$W(k)$	wavenumber response, (2)
$F(k)$	auxiliary function, (2)
$Y(k)$	auxiliary function, (2)
k_L	wavenumber defined by periodic spacing, (2)
B_q	dynamic structural mass of q-th rib stiffener set, (2)
Δ_q	q-th offset distance, (2)
F_o	magnitude of applied line force, (3)
x_o	point of application of line force, (3)
$S(k)$	auxiliary function, (4)
k_b	free in-vacuum plate wavenumber, (4)
ρ_o	mass density of acoustic fluid, (4)
k_o	acoustic wavenumber ω/c_o, (4)
m'_q	mass per unit length of q-th set of rib stiffeners

m,n,p,q	integers
ε	real nonzero variable, (5)
$\{A_n\}$	given sequence, (5)
α_q	q-th exponential factor, (6)
$\underline{W}_q(k)$	auxiliary summation function, (8)
$\underline{F}_q(k)$	auxiliary summation function, (13)
$\underline{Y}_{pq}(k)$	auxiliary summation function, (14)
N(k)	numerator term for Q = 2, (20)
D(k)	denominator term for Q = 2, (21)
B(u)	Fourier sum, (22)
$\underline{W}(u,k)$	auxiliary summation function, (24)
$\underline{F}(u,k)$	auxiliary summation function, (29)
$\underline{Y}(v-u,k)$	auxiliary summation function, (30)
$\underline{W}_p(u,k)$	p-th order approximation to $\underline{W}(u,k)$, (33)

EXPLICIT SOLUTION OF DIFFERENCE EQUATION FOR THE WAVENUMBER RESPONSE OF FLUID-LOADED STIFFENED PLATE

INTRODUCTION*

Acoustic radiation from the upper surface of a fluid-loaded stiffened plate has been examined by Cray [1]. The formulation developed there produces an implicit algebraic equation for the Fourier transformed plate wavenumber response. This response must be manipulated into an explicit form in order to obtain a solution for the plate's structural response and to obtain the near- and far-field generated acoustic response. The following is a brief synopsis of the formulation for the acoustic radiation from a stiffened plate.

The infinite plate investigated in [1] was configured to have two infinite sets of attached rib stiffeners. The stiffeners composing a given set were identical and were spaced periodically with distance L. One set of stiffeners, however, was shifted by an amount Δ from the other set. In this manner, portions of the plate were configured with repeating sections having nonperiodic rib spacing.

The governing equation of motion for the surface displacement of the fluid-loaded isotropic plate for an applied line-force, with Q sets of attached rib stiffeners, is given by

$$D\,\frac{d^4 w(x)}{dx^4} - m\,\omega^2\,w(x) = P_e(x) - P_a(x,0) - \sum_{q=1}^{Q} P_q(x)\;, \qquad (1)$$

*The introduction was contributed by Dr. Benjamin A. Cray.

where D is the rigidity of the plate, $w(x)$ is the transverse plate displacement, m is the mass per unit area of the plate, ω is the applied excitation frequency, $P_e(x)$ is the external pressure due to the applied line-force, $P_a(x,0)$ is the acoustic pressure acting on the upper surface of the plate, and $\{P_q(x)\}$ are the total pressures exerted by each set of attached rib stiffeners.

Rotary inertia and shear deformation effects within the plate and rib stiffeners were neglected. The stiffeners exerted reactive forces upon the plate but applied no angular moments.

Equation (1) was transformed, term by term, using exponential Fourier transforms. The spatial transform variable, k, has the physical significance of wavenumber. It was assumed that the transforms are well defined and exist over the entire domain of integration.

Upon transforming (1) into the wavenumber domain, the implicit form of the wavenumber response, for the fluid-loaded stiffened plate, can be written as

$$W(k) = F(k) - Y(k) \sum_{n=-\infty}^{\infty} W(k + nk_L) \times$$
$$\times \left(B_1 + B_2 \exp\left(ink_L\Delta_1\right) + \cdot \cdot \cdot + B_Q \exp\left(ink_L\Delta_{Q-1}\right)\right) , \qquad (2)$$

where $W(k)$ is the (transformed) wavenumber response. The quantities on the right-hand side of (2) are defined as

$$F(k) = \frac{F_0 \exp(-ikx_0)}{S(k)} , \quad Y(k) = \frac{1}{S(k)} , \qquad (3)$$

where

$$S(k) = \begin{cases} D\left(k^4 - k_b^4\right) - \dfrac{i\ \rho_o\ \omega^2}{\sqrt{k_o^2 - k^2}} & \text{for } |k| < k_o \\ D\left(k^4 - k_b^4\right) - \dfrac{\rho_o\ \omega^2}{\sqrt{k^2 - k_o^2}} & \text{for } |k| > k_o \end{cases}, \tag{4}$$

where $k_b = (m\ \omega^2/D)^{1/4}$ is the free in-vacuum plate wavenumber, $k_L = 2\pi/L$ is the wavenumber defined by the periodic spacing, and $B_q = m_q'\ \omega^2/L$ is the dynamic structural mass of the q-th rib stiffener set. Also, Δ_q is the q-th offset distance.

Notice in (2) that the wavenumber or spectral response W(k) appears implicitly. The difficulty now lies in determining an explicit expression for the plate's wavenumber response W(k). That is, it is necessary to manipulate (2) such that the summation which contains the shifted wavenumber responses $\{W(k + nk_L)\}$ may be rewritten in terms of known quantities.

In [1], Cray obtained an explicit expression for W(k) for the case of a single offset, that is, Q = 2 in (2). This corresponds to two sets of rib stiffeners, where one set is offset from the other. The following development is a significant extension of the configuration of two sets of rib stiffeners to the case of an arbitrary number of rib stiffener sets, each with different offsets $\{\Delta_q\}$, and to arbitrary excitation functions.

DESCRIPTION AND SOLUTION OF DIFFERENCE EQUATION

In the following mathematical development, k and ε are arbitrary real variables, while m,n,p,q are integers. Also, summations without limits are from $-\infty$ to $+\infty$. We are interested in finding the solution $W(k)$ to the difference equation

$$W(k) = F(k) - Y(k) \sum_n A_n \, W(k + \varepsilon n) \, , \tag{5}$$

where $F(k)$ and $Y(k)$ are given known functions of k, and sequence $\{A_n\}$ is also known. For $\varepsilon = 0$, the solution to (5) is immediate; hence, ε is nonzero and fixed in the following.

In fact, we are interested in the particular case where A_n is a finite sum of Q complex exponentials in n (compare with the second line of (2)):

$$A_n = \sum_{q=1}^{Q} B_q \exp(in\alpha_q) \quad \text{for all } n \, , \tag{6}$$

where $\{\alpha_q\}$, $1 \le q \le Q$, are arbitrary (complex) <u>distinct</u> constants, and $\{B_q\}$ are arbitrary complex constants. The $\{B_q\}$ and $\{\alpha_q\}$ are given. Furthermore, since

$$\exp(in\alpha_q) = \exp(in[\alpha_q - p2\pi]) \quad \text{for all } p \, , \tag{7}$$

there is <u>no</u> <u>loss</u> of generality in assuming that $|\mathrm{Re}(\alpha_q)| \le \pi$ for all $1 \le q \le Q$. If $\mathrm{Im}(\alpha_q)$ is nonzero for any q, then $|A_n|$ cannot remain bounded at both limits of $n = \pm\infty$. The earlier case solved in [1] corresponded to $Q = 2$ and the special value of $\alpha_1 = 0$.

DEFINITION AND PROPERTIES OF AUXILIARY FUNCTIONS

Define functions

$$\underline{W}_q(k) = \sum_n W(k + \varepsilon n) \exp(in\alpha_q) \quad \text{for } 1 \le q \le Q . \tag{8}$$

(The additional dependence of $\underline{W}_q(k)$ on ε is suppressed notationally.) It then follows that

$$\underline{W}_q(k + \varepsilon m) = \sum_n W(k + \varepsilon m + \varepsilon n) \exp(in\alpha_q) =$$

$$= \sum_p W(k + \varepsilon p) \exp(i(p-m)\alpha_q) = \exp(-im\alpha_q)\, \underline{W}_q(k) , \tag{9}$$

where we let $p = m + n$ and used (8). This is the key relation regarding the functions $\underline{W}_q(k)$ defined in (8), namely

$$\underline{W}_q(k + \varepsilon m) = \exp(-im\alpha_q)\, \underline{W}_q(k) \quad \text{for } 1 \le q \le Q . \tag{10}$$

With the aid of (6) and (8), the summation on n in (5) can now be manipulated into the form

$$\sum_n A_n W(k + \varepsilon n) = \sum_n W(k + \varepsilon n) \sum_{q=1}^{Q} B_q \exp(in\alpha_q) = \sum_{q=1}^{Q} B_q \underline{W}_q(k) . \tag{11}$$

This enables us to express (5) in the alternative form

$$W(k) = F(k) - Y(k) \sum_{q=1}^{Q} B_q \underline{W}_q(k) . \tag{12}$$

SOLUTION OF EQUATION (12)

At this point, in analogy to (8), it is convenient to define two additional functions, namely,

$$\underline{F}_q(k) = \sum_n F(k + \varepsilon n) \exp(in\alpha_q) \quad \text{for } 1 \le q \le Q \ , \tag{13}$$

$$\underline{Y}_{pq}(k) = \sum_n Y(k + \varepsilon n) \exp\left(in(\alpha_p - \alpha_q)\right) \quad \text{for } 1 \le p,q \le Q \ . \tag{14}$$

Now, replace k by $k + \varepsilon m$ in (12) and use (10), obtaining

$$W(k + \varepsilon m) = F(k + \varepsilon m) - Y(k + \varepsilon m) \sum_{q=1}^{Q} B_q \exp(-im\alpha_q) \, \underline{W}_q(k) \ . \tag{15}$$

When this equation is multiplied by $\exp(im\alpha_p)$ and summed over all m, there follows, by use of (8), (13), and (14),

$$\underline{W}_p(k) = \underline{F}_p(k) - \sum_{q=1}^{Q} \underline{Y}_{pq}(k) \, B_q \, \underline{W}_q(k) \quad \text{for } 1 \le p \le Q \ . \tag{16}$$

This relation constitutes Q linear equations (at each k) in the Q unknown functions $\underline{W}_q(k)$, $1 \le q \le Q$. The remaining quantities in (16) can be obtained from (6), (13), and (14).

When (16) is solved for all the $\{\underline{W}_q(k)\}$, then (12) directly yields the original quantity of interest, namely W(k) in (5), for that <u>particular</u> value of k. Although this procedure appears to require the solution of a new set of Q simultaneous linear equations for each k of interest, there is a shortcut that might be useful in some cases. Namely, reference to (15) reveals that

$W(k + \varepsilon m)$ can now be calculated easily for nonzero m, for <u>that</u> k, once $F(k + \varepsilon m)$ and $Y(k + \varepsilon m)$ have been computed, without the need for another solution set. The <u>same</u> solutions $\{\underline{W}_q(k)\}$ are used on the right-hand side of (15), regardless of the value of m under consideration; only the finite summation in (15) need be redone. Thus, the solution W to (5) at arguments k, $k \pm \varepsilon$, $k \pm 2\varepsilon, \ldots$ can be found from the solution of one set of Q simultaneous equations, (16).

For the special case of $Q = 1$, (16) can be immediately solved to yield [1]

$$\underline{W}_1(k) = \frac{\underline{F}_1(k)}{1 + B_1\, \underline{Y}_{11}(k)} = \frac{\sum_n F(k + \varepsilon n) \exp(in\alpha_1)}{1 + B_1 \sum_n Y(k + \varepsilon n)} . \qquad (17)$$

Then (12) yields the explicit result (for $Q = 1$)

$$W(k) = F(k) - B_1\, Y(k)\, \frac{\sum_n F(k + \varepsilon n) \exp(in\alpha_1)}{1 + B_1 \sum_n Y(k + \varepsilon n)} . \qquad (18)$$

In this form, it is possible to directly evaluate W(k) at any k of interest merely by computing the terms encountered on the right-hand side of (18). However, shortcut (15) is still better for the particular arguments $\{k + \varepsilon m\}$.

For Q = 2, when the solutions of (16) are substituted into (12), the function W(k) is given by the explicit expression

$$W(k) = F(k) - Y(k)\,\frac{N(k)}{D(k)}\,, \tag{19}$$

where

$$N(k) = B_1\,\underline{F}_1(k) + B_2\,\underline{F}_2(k) + B_1\,B_2\,\times$$
$$\times\left[\underline{F}_1(k)\,\underline{Y}_{22}(k) + \underline{F}_2(k)\,\underline{Y}_{11}(k) - \underline{F}_1(k)\,\underline{Y}_{21}(k) - \underline{F}_2(k)\,\underline{Y}_{12}(k)\right] \tag{20}$$

and

$$D(k) = 1 + B_1\,\underline{Y}_{11}(k) + B_2\,\underline{Y}_{22}(k) +$$
$$+ B_1\,B_2\left[\underline{Y}_{11}(k)\,\underline{Y}_{22}(k) - \underline{Y}_{12}(k)\,\underline{Y}_{21}(k)\right]\,. \tag{21}$$

A program for the evaluation of (19) - (21), for the case where Y(k) is real, is presented in the appendix; also, B_1, B_2, α_1, and α_2 are real. Then D(k) is real, but N(k), F(k), and W(k) are complex. Also, $\underline{Y}_{22}(k) = \underline{Y}_{11}(k)$ is real, while $\underline{F}_1(k)$, $\underline{F}_2(k)$, and $\underline{Y}_{21}(k)^* = \underline{Y}_{12}(k)$ are complex. The program uses these properties.

In general, for any Q, if (16) is solved <u>analytically</u> for the set $\{\underline{W}_q(k)\}$ in terms of $\{B_q\}$, $\{\underline{F}_q(k)\}$, and $\{\underline{Y}_{pq}(k)\}$, these results can be substituted or utilized in (12) to get an explicit expression for W(k) that is valid for all k. For large Q, this will be impractical; instead, numerical solution of the Q simultaneous equations, (16), will be required at each k of interest. The only exception is to use (15) for the particular arguments $\{k + \varepsilon m\}$. (All results above actually hold for k and ε complex.)

SOLUTION FOR GENERAL SEQUENCE $\{A_n\}$

In this section, we no longer restrict sequence $\{A_n\}$ to have exponential form (6). Rather, for bounded given sequence $\{A_n\}$, define function

$$B(u) = \frac{1}{2\pi} \sum_n A_n \exp(-inu) \quad \text{for } |u| < \pi \ , \tag{22}$$

where u is real. Then, the sequence values can be found from

$$A_n = \int_{-\pi}^{\pi} du\ B(u) \exp(inu) \quad \text{for all } n \ . \tag{23}$$

The fastest variation with n that (23) allows for A_n is $\exp(\pm in\pi)$; however, this is no loss of generality, as seen by reference to (7). The case considered earlier in (6) corresponds to B(u) being composed of a set of Q impulses of area B_q located at $u = \alpha_q$, with $|\alpha_q| \le \pi$, when α_q is real.

Now, define function

$$\underline{W}(u,k) = \sum_n W(k + \varepsilon n) \exp(inu) \quad \text{for } |u| \le \pi \ . \tag{24}$$

(Again, the notational dependence on ε is suppressed.) The key relation that $\underline{W}(u,k)$ satisfies is

$$\underline{W}(u,k + \varepsilon m) = \sum_n W(k + \varepsilon m + \varepsilon n) \exp(inu) =$$

$$= \sum_p W(k + \varepsilon p) \exp(i(p-m)u) = \exp(-imu)\ \underline{W}(u,k) \ , \tag{25}$$

where we let $p = m + n$ and used (24). This relation,

$$\underline{W}(u,k + \varepsilon m) = \exp(-imu)\ \underline{W}(u,k) \ , \tag{26}$$

is the analogue of (10) earlier.

The summation in (5) can now be expressed as

$$\sum_n A_n\, W(k + \varepsilon n) = \sum_n W(k + \varepsilon n) \int_{-\pi}^{\pi} du\ B(u) \exp(inu) =$$

$$= \int_{-\pi}^{\pi} du\ B(u)\ \underline{W}(u,k) \ , \tag{27}$$

where we used (23) and (24). Therefore, the equation of interest, (5), now takes the form

$$W(k) = F(k) - Y(k) \int_{-\pi}^{\pi} du\ B(u)\ \underline{W}(u,k) \ . \tag{28}$$

We now define, in analogy to (13) and (14), the two functions

$$\underline{F}(u,k) = \sum_n F(k + \varepsilon n) \exp(inu) \quad \text{for } |u| \le \pi \ , \tag{29}$$

$$\underline{Y}(v\text{-}u,k) = \sum_n Y(k + \varepsilon n) \exp(in(v-u)) \quad \text{for } |u|,|v| \le \pi \ . \tag{30}$$

When we replace k by $k + \varepsilon m$ in (28), and use (26), there follows

$$W(k + \varepsilon m) = F(k + \varepsilon m) - Y(k + \varepsilon m) \int_{-\pi}^{\pi} du\ B(u)\ \underline{W}(u,k) \exp(-imu) \ . \tag{31}$$

We now multiply (31) by exp(imv), v real, and sum over all m, to

obtain

$$\underline{W}(v,k) = \underline{F}(v,k) - \int_{-\pi}^{\pi} du\ \underline{Y}(v-u,k)\ B(u)\ \underline{W}(u,k) \quad \text{for } |v| \le \pi\ , \tag{32}$$

where we used (24), (29), and (30). This is a linear integral equation, with kernel $\underline{Y}(v-u,k)\ B(u)$, for unknown $\underline{W}(u,k)$; that is, for each k of interest, a new linear integral equation must be solved. Then, $\underline{W}(u,k)$ will be known for $|u| \le \pi$ at that particular k.

Once $\underline{W}(u,k)$ is known, (28) gives desired solution W(k) at that particular k value, since F(k), Y(k), and B(u) are known functions. If a large number of k values are of interest, (32) can involve a great deal of computational effort; however, arbitrary bounded sequence $\{A_n\}$ is now allowed in (5). Furthermore, (31) then yields the solution for W at arguments $k,\ k \pm \varepsilon,\ k \pm 2\varepsilon, \ldots$ without the need to solve another integral equation.

The linear integral equation in (32) could be solved by recursion; for example, the p-th order approximation is given by

$$\underline{W}_p(v,k) \equiv \underline{F}(v,k) - \int_{-\pi}^{\pi} du\ \underline{Y}(v-u,k)\ B(u)\ \underline{W}_{p-1}(u,k) \quad \text{for } |v| \le \pi\ . \tag{33}$$

A possible starting value for this recursion is $\underline{W}_o(u,k) = \underline{F}(u,k)$.

SUMMARY

When the governing differential equation of motion for the surface displacement of the fluid-loaded isotropic plate for an applied line-force, with Q sets of attached rib stiffeners, is transformed into the wavenumber domain, a difference equation is encountered. Solution of this latter equation for exponential excitation, for a particular value of wavenumber k, is accomplished through the definition of auxiliary functions involving sums of displaced wavenumber responses with exponential factors. Finally, solution of a simultaneous set of Q linear equations completes the required calculations. For the most general excitation, the set of equations becomes infinite, that is, a linear integral equation must be solved.

APPENDIX. PROGRAM FOR (19) - (21)

In the following program for solution W(k) given by (19) - (21), specified function Y(k) is presumed real; however, F(k) is allowed to be complex. The following example corresponds to

$$F(k) = \exp(-.71\ k^2) + i\ \exp(-.91\ k^2 - k)\ ,$$

$$Y(k) = \exp(-.63\ k^2 + k)\ .$$

```
 10     K=.7                      !  k, wavenumber
 20     E=.4                      !  epsilon
 30     L1=.1                     !  alpha1
 40     L2=.2                     !  alpha2
 50     B1=1.1
 60     B2=1.3
 70     CALL Fqk(K,E,L1,L2,F1r,F1i,F2r,F2i)      ! (13)
 80     CALL Ypqk(K,E,L1,L2,Y11,Y12r,Y12i)       ! (14)
 90     B=B1*B2
100     Y=Y11-Y12r
110     D=Y11*Y11-Y12r*Y12r-Y12i*Y12i
120     D=1.+(B1+B2)*Y11+B*D                     ! (21)
130     Nr=(F1r+F2r)*Y-(F1i-F2i)*Y12i
140     Nr=B1*F1r+B2*F2r+B*Nr                    ! (20)
150     Ni=(F1i+F2i)*Y+(F1r-F2r)*Y12i
160     Ni=B1*F1i+B2*F2i+B*Ni                    ! (20)
170     PRINT
180     PRINT "DENOMINATOR = ";D
190     Y=FNY(K)/D
200     CALL F(K,Fr,Fi)
210     Wr=Fr-Y*Nr
220     Wi=Fi-Y*Ni                               ! (19)
230     PRINT Wr,Wi
240     END
250     !
```

```
260     SUB Fqk(K,E,L1,L2,F1r,F1i,F2r,F2i)          ! (13)
270     DOUBLE N            !  INTEGER
280     F1r=F1i=F2r=F2i=0.
290     FOR N=-40 TO 40
300     CALL F(K+E*N,Fr,Fi)
310     IF ABS(N)=40 THEN PRINT ABS(Fr)+ABS(Fi);
320     C=COS(N*L1)
330     S=SIN(N*L1)
340     F1r=F1r+Fr*C-Fi*S
350     F1i=F1i+Fr*S+Fi*C
360     C=COS(N*L2)
370     S=SIN(N*L2)
380     F2r=F2r+Fr*C-Fi*S
390     F2i=F2i+Fr*S+Fi*C
400     NEXT N
410     SUBEND
420     !

430     SUB Ypqk(K,E,L1,L2,Y11,Y12r,Y12i)           ! (14)
440     DOUBLE N            !  INTEGER
450     Y11=Y12r=Y12i=0.
460     FOR N=-40 TO 40
470     T=FNY(K+E*N)
480     IF ABS(N)=40 THEN PRINT T;
490     A=N*(L1-L2)
500     Y11=Y11+T
510     Y12r=Y12r+T*COS(A)
520     Y12i=Y12i+T*SIN(A)
530     NEXT N
540     SUBEND
550     !
560     SUB F(K,Fr,Fi)                              ! F(k)
570     Fr=Fi=0.
580     A=.71*K*K
590     IF A>100. THEN 610
600     Fr=EXP(-A)
610     A=.91*K*K+K
620     IF A>100. THEN 640
630     Fi=EXP(-A)
640     SUBEND
650     !
660     DEF FNY(K)                                  ! real Y(k)
670     A=.63*K*K-K
680     IF A>100. THEN RETURN 0.
690     RETURN EXP(-A)
700     FNEND
```

REFERENCES

[1] B. A. Cray, **Near-Field and Far-Field Sound Radiation from a Line-Driven Fluid-Loaded Infinite Flat Plate Having Periodic and Nonperiodic Attached Rib Stiffeners**, NUWC-NL Technical Report TR 10013, Naval Undersea Warfare Center, New London, CT, March 1992.

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